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Authors: J. E Benmansour\(^{(1)}\), b. Khouane\(^{(2)}\), R. Rima\(^{(1)}\)  
Affiliations:  
1. Département de Recherche en Mécanique Spatiale Centre de Développement des Satellites (CDS) Oran, Algeria  
2. Département de Mission et Systèmes Spatiaux Centre de Développement des Satellites (CDS) Oran, Algeria  
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High Precision Roll/Yaw Attitude Stabilization for Flexible communication satellite

J. E Benmansour\textsuperscript{(1)*}, b. Khouane\textsuperscript{(2)}, R. Rima\textsuperscript{(1)}

\textsuperscript{(1)} Département de Recherche en Mécanique Spatiale Centre de Développement des Satellites (CDS) Oran, Algeria
\textsuperscript{(2)} Département de Mission et Systèmes Spatiaux Centre de Développement des Satellites (CDS) Oran, Algeria
\textsuperscript{*}jebenmansour@cds.asal.dz

Abstract: The aim of this paper is to realize high-precision attitude stabilization for roll/yaw axes of flexible communication satellite while attenuate the effects of the elastic vibrations and multiple disturbances such as solar radiation and model uncertainties. A composite control has been designed which comprise two part an anti-disturbance proportional-derivative (PD) controller is designed to stabilize the attitude while rejecting the effects of flexible vibrations, environmental disturbances, and unmodelled dynamics, whose are assumed as an extended state. This controller comprises two parts, i.e. an extended state observer and a PD controller with feedforward. First, flexible vibrations, environmental disturbances and unmodelled dynamics are regarded as an extended state, which can be estimated by the proposed observer. The estimated extended state can be compensated by feedforward where the attitude can be stabilized by the PD controller. Numerical simulation results are presented to demonstrate the effectiveness of the control scheme.

Keywords: Flexible spacecraft; Attitude stabilization; Extended state observer; Proportional Derivative

1. INTRODUCTION
The attitude control system is an important topic in spacecraft missions while it keeps the body in designed orbit and desired attitude. Both control elements and control algorithms are developed for the control missions by control engineers [1]. Early attitude control for spacecraft was based on the assumption that the body was rigid. However, the new communication satellites are tricky in a control point of view because their conceptions present several sources of flexibilities: solar arrays, antennas that can be very large with respect to the main body, which seriously affect attitude control performance [2, 3].

Over the last decades, many researchers have conducted extensive studies on the spacecraft attitude control system since for flexible dynamics the task is more delicate since the flexible modes which are close to the control bandwidth can lead to lose stability and it is difficult for PID/PD controller to get satisfactory performance for the communication satellite.

Early in the 1980s, an optimal control scheme has been investigated of flexible spacecraft for vibration suppression problems [4,5]. Afterward the sliding mode control, which is a very effective approach and has been applied to attitude control system design by combining with active vibration control [6-7]. However this technique often results in chattering due to its discontinuous switching controller. In [8-9] the studies were carried out for H\textsuperscript{\infty} control in based on the linear matrix inequality, which was designed for flexible spacecraft and could provide a good disturbance attenuation performance, but it was of limited usefulness for attitude stabilization and active vibration control. An adaptive attitude controllers was also applied to address the tracking problem of spacecraft in the presence of unknown control input saturation and external disturbances [10,11].

To this end, the development of disturbance estimation techniques would be a good choice to alleviate the restriction faced by traditional feedforward control. In the past few years, the disturbance observer-based control has been regarded as one of the most promising disturbance rejection; it may have less conservativeness for many types of disturbances and is easy to integrate with other feedback controllers: PD, H\textsuperscript{\infty} and variable structure controllers[12-15].

Unlike to [17] where only one axis has been taking account in this paper, a composite controller design approach is put forward for Roll/Yaw attitude flexible spacecraft based on extended state observer and PD controller. The extended state observer can reject the effect of vibrations from...
flexible appendages, and the PD controller can attenuate the effect of the disturbances and model uncertainties, assuming the disturbances as an extended state. It is shown that, The composite attitude control scheme improves the accommodation of uncertainties and unknown disturbances to give the spacecraft attitude control system high precision and high stability, thus by properly choosing the disturbance compensation gains, the uncertainties can be attenuated from the system output. Simulations for a flexible spacecraft show that the performance of the composite attitude control system can be improved by the proposed method compared with pure PD control.

2. MODEL DESCRIPTION
In order to simplify the model, the satellite is modelled as a main rigid body with flexible appendages, as shown in Figure 1.

![Fig. 1: Satellite with flexible appendages](image)

This model can be derived from the non-linear attitude dynamics of the spacecraft. As studied in [14-16]

\[
\begin{align*}
I_{xx} \ddot{\phi} + \alpha_h \theta \phi + h \theta \psi + F_x^T \dot{\eta} &= u_x + d_{x0} \\
I_{zz} \ddot{\psi} + \alpha_h \theta \psi + h \theta \phi + F_z^T \dot{\eta} &= u_z + d_{z0} \\
\dot{\eta} + 2 \zeta \Omega \eta + \Omega^2 \eta + F_x \dot{\phi} + F_z \dot{\psi} &= 0
\end{align*}
\]

(1)

where \(I_{xx}, I_{zz}\) are the principals moments of inertia of the spacecraft; \(\phi, \psi\) are yaw and roll angles; \(\alpha_h\) is the angular velocity vector in the body-fixed frame with respect to the inertial frame; \(h\) is the nominal wheel momentum; \(d_{x0}, d_{z0}\) are the disturbances; \(u_x, u_z\) are the roll and yaw commands, respectively. The third equation of system (1) describes the dynamics of the flexible model, and \(\Omega\) is the model frequency, \(F\) is the coupling matrix, \(\zeta\) is the damping ratio and \(\eta\) is the flexible model coordinates.

Let

\[
\begin{align*}
\dot{d}_x &= d_{x0} - F_x^T \dot{\eta} \\
\dot{d}_z &= d_{z0} - F_z^T \dot{\eta}
\end{align*}
\]

(2)

Then, the system (1) can be formulated as:

\[
\begin{align*}
I_{xx} \ddot{x}_1 + \alpha_h \theta \phi + h \theta \psi &= u_x + d_x \\
I_{zz} \ddot{x}_2 + \alpha_h \theta \psi + h \theta \phi &= u_z + d_z
\end{align*}
\]

(3)

Denote \(x_1 = \phi, x_2 = \psi, x_3 = \dot{\phi}, x_4 = \dot{\psi}\),

\[
X = (x_1, x_2, x_3, x_4)^T, \quad u = (u_x, u_z)^T, \quad d = (d_x, d_z)^T.
\]
Then system (3) can be transformed into the following form:

\[ \dot{X} = Ax + B(u + d) \quad (4) \]

where

\[
A = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\frac{-\omega h_0}{l_{xx}} & 0 & 0 & \frac{h_0}{l_{xx}} \\
\frac{-\omega h_0}{l_{zz}} & 0 & 0 & \frac{h_0}{l_{zz}}
\end{bmatrix},
B = \begin{bmatrix}
0 & 0 \\
1 & 0 \\
\frac{1}{l_{xx}} & 0 \\
0 & \frac{1}{l_{zz}}
\end{bmatrix}.
\]

Define \( x_5 = d_x \), \( x_6 = d_z \), \( X_a = (X^T, x_5, x_6)^T \) and

\[
A_a = \begin{bmatrix}
A & B \\
0_{2 \times 4} & 0_{2 \times 2}
\end{bmatrix}, B_a = \begin{bmatrix}
B \\
0_{2 \times 2}
\end{bmatrix}
\]

Eq. (4) can be extended to the following equation:

\[ \dot{X}_a = A_a X_a + B_a u \quad (5) \]

If both roll and yaw attitude are measured, then the output equation can be written as:

\[ y = CX_a \quad (6) \]

where

\[
C = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0
\end{bmatrix}, \quad y = (y_1, y_2)
\]

Examine the matrix \( N = [C, CA, \ldots, CA^5]^T \), it is easy to verify that \( \text{rank}(N) = 6 \), therefore system (5), (6) is observable.

Assume that \( \hat{X}_a \) is the estimate of \( X_a \), then a state observer for (4) and (5) can be designed as:

\[
\begin{align*}
\dot{\hat{X}}_a &= A_a \hat{X}_a + B_a u + L(y - \hat{y}) \\
\dot{\hat{y}} &= C \hat{X}_a
\end{align*}
\quad (7)
\]

where \( L \) is the gain matrix.

Substitute \( \dot{\hat{y}} = C \hat{X}_a \) into (7), then:

\[
\dot{\hat{X}}_a = A_a \hat{X}_a + B_a u + LC \left( X_a - \hat{X}_a \right) \quad (8)
\]

The estimation error \( \hat{X}_a = X_a - \hat{X}_a \), then the error dynamics can be derived from (5) and (8) as follows:

\[
\dot{\hat{X}}_a = (A_a - LC) \hat{X}_a \quad (9)
\]

Since the pair \((A_a, LC)\) is observable, there exists gain matrix \( L \) so that \((A_a - LC)\) is Hurwitz. Furthermore, all eigenvalues can be placed in the designated places of the left half plane through the proper choice of the gain matrix \( L \), since all eigenvalues of \((A_a - LC)\) can have negative real parts, the error dynamics (9) is stable asymptotically.
3. COMPOSITE CONTROL DESIGN

The composite controller can be described by Figure 2 and it has two parts: a time-domain disturbance observer and a state feedback controller. The First loop is used to estimate flexible vibrations, environmental disturbances, which are regarded as an extended state. Then, the estimated extended state can be compensated by feedforward and the second loop provides the PD controller to stabilize the attitude.

Examine the matrix \( M = \begin{bmatrix} B & AB & A^2B & A^3B \end{bmatrix} \), it is easy to verify that rank \((M) = 4\), then system (4) is controllable.

The controller can be described as

\[ u = -K\hat{X} - \hat{d} \]  

where \( \hat{X} = (\hat{x}_1 \quad \hat{x}_2 \quad \hat{x}_3 \quad \hat{x}_4)^T \), \( \hat{d} = (\hat{d}_5 \quad \hat{d}_6) \), \( K \) is the gain matrix, \( \phi_d, \psi_d, \phi_d, \psi_d \) are the desired attitude and \( \hat{d} \) is the estimation of \( d \).

The performance of the controller can be analysed as follows. Substitute (10) into (4), then:

\[ \dot{X} = AX - BK\hat{X} + Bd \]  

where \( \dot{d} = d - \hat{d} \).

While the estimated state \( \hat{X} = X - \bar{X} \), and then Eq. (11) can be rewritten as:

\[ \dot{X} = (A - BK)X + BK\bar{X} + Bd \]  

Since system (4) is controllable, \( (A - BK) \) is Hurwitz and its eigenvalues can be placed in the expected locations in the left half plane by choosing the gain matrix \( K \). However, Hurwitz matrix \( (A - BK) \) cannot guarantee the whole closed-loop stability due to the existence of \( \dot{X} \) and \( \dot{d} \) in (12).

To analyze the stability of the closed-loop system, we must combine the states \( X, \bar{X} \), together. According to (9) and (12), the closed-loop error dynamics is governed by:

\[ \begin{bmatrix} \dot{X} \\ \dot{X}_a \end{bmatrix} = \begin{bmatrix} A - BK & BK \\ O_{6 \times 4} & A_a - LC \end{bmatrix} \begin{bmatrix} X \\ X_a \end{bmatrix} \]  

where \( O_{6 \times 4} \) is a zero matrix with dimensions \( 6 \times 4 \).

According to the matrix theory, the eigenvalues of \( \begin{bmatrix} A - BK & BK \\ O_{6 \times 4} & A_a - LC \end{bmatrix} \) are the same as \( (A - BK) \) and \( (A_a - LC) \) which all can have negative real parts through the proper design of gain matrix \( K \) and \( L \). However, the controller gains and the observer gains can be calculated based on the
characteristic equations of \((A - BK)\) and \((A_u - LC)\) respectively via method of undetermined coefficients, such as:

\[
\lambda E_1 - (A - BK) = 0 \quad (14)
\]

and

\[
\lambda E_2 - (A_u - LC) = 0 \quad (15)
\]

where \(E_1\) is a \(3 \times 3\) unit matrix and \(E_2\) is a \(4 \times 4\) unit matrix.

From this point, it can be concluded that the closed-loop error system (13) is stable asymptotically.

4. NUMERICAL SIMULATIONS

The following results help explain the proposed control schemes, such as the composite control scheme has been applied to a satellite with one flexible appendage. It is supposed that the flexible modes are \(\omega_z = \omega_x = 0.5\) rad/s with the damping ratio \(\xi_z = 0.05\) and \(\xi_x = 0.04\), while the moment of inertia of the Roll and Yaw are respectively \(I_{xx} = 3026\,\text{Kgm}^2\), \(I_{zz} = 3146\,\text{Kgm}^2\) and the coupling coefficient matrix is \(F = \begin{bmatrix} 36 & 35 \\ 35 & 36 \end{bmatrix}\), the matrix of controller gain is designed as

\[
K = \begin{bmatrix} 12 & 0 & 72 & 0 \\ 0 & 12 & 0 & 72 \end{bmatrix}
\]

and the matrix of observer gain is \(L = \begin{bmatrix} 2.5 & 0 & 1.56 & 0 & 6050 & 0 \\ 0 & 2.5 & 0 & 2.86 & 0 & 7030 \end{bmatrix}^T\).

The flexible spacecraft is designed to move in the geostationary orbit with the altitude of 36000 Km, then the orbit rate \(n = 7.292 \times 10^{-5}\) rad/s. The external disturbances torques acting on the satellite is assumed as

\[
\begin{align*}
T_x &= 2 \times 10^{-5} (1 - 2 \sin \omega_x t) \text{Nm} \\
T_z &= -5 \times 10^{-5} (\cos \omega_x t) \text{Nm}
\end{align*} \quad (16)
\]

The initial attitudes of the spacecraft are \([\theta \ \psi] = [4.5 \ 5]\) deg, initial attitude rate are \([\dot{\theta} \ \dot{\psi}] = [0.05 \ 0.05]\) deg/s, and simulation results are shown in Figs. 2-6, where we denote the abbreviation ‘ESO’ for the extended state observer.

![Fig. 3: Time responses of Roll angle](image-url)
Under the same simulation conditions, Figs 3 and 4 show the time response of Roll and Yaw angle, it is clear that the attitude angles have fine dynamic response performance controlled by PD including ESO compared with pure PD, such as the attitude control accuracy is improved by the composite controller, which is clearly increased than the PD.

Figs 5 and 6 show the time response of elastic vibration, vibration observer and estimation error, where one can be seen that the vibration from the flexible appendages can be estimated by the
disturbance observer. Thus, with the estimation, the effect of the elastic vibration to the rigid body is reduced by feed-forward composition.

For more detailed analysis, the root mean square (RMS) values of error results are computed for the time period 500-700 sec, which are presented in the following Table.

<table>
<thead>
<tr>
<th>Controller</th>
<th>RMS of attitude (deg)</th>
<th>RMS of attitude rate (deg/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PD Controller</td>
<td>Roll 0.0142</td>
<td>Roll 0.001</td>
</tr>
<tr>
<td></td>
<td>Yaw 0.02</td>
<td>Yaw 0.0012</td>
</tr>
<tr>
<td>PD+ESO controller</td>
<td>Roll 0.005</td>
<td>Roll 2.63e-04</td>
</tr>
<tr>
<td></td>
<td>Yaw 0.0065</td>
<td>Yaw 3.94e-4</td>
</tr>
</tbody>
</table>

5. CONCLUSIONS
In this paper, a composite control approach combining the PD with the ESO is proposed for the attitude control system of flexible spacecraft to enhance disturbance attenuation ability and robust performance; it has been shown how extended state observer is used for attitude control of communication satellite, where the extended state observer can estimate flexible vibrations, and
environmental disturbances such as solar pressure. These disturbances are compensated by feed-forward while the PD controller can stabilize the attitude afterwards. Numerical simulations have shown that the pointing accuracy and the stabilization of the spacecraft can be improved by the composite controller compared with the pure PD control. This method provides a useful and promising way for the attitude control of flexible spacecraft since it can be applied in real systems. For future research, it is required to take into account the fuel consumption minimization problem by extending models and designs a new composite controller to achieve better control effect.

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**References**